ARTICLE NO. 79-0079R

Efficient Algorithm for Numerical Airfoil Optimization

Garret N. Vanderplaats*
NASA Ames Research Center, Moffett Field, Calif.

A new optimization algorithm is presented. The method is based on sequential application of a second-order Taylor's series approximation to the airfoil characteristics. Compared to previous methods, design efficiency improvements of more than a factor of 2 are demonstrated. If multiple optimizations are performed, the efficiency improvements are more dramatic due to the ability of the technique to utilize existing data. The method is demonstrated by application to subsonic and transonic airfoil design but is a general optimization technique and is not limited to a particular application or aerodynamic analysis.

Introduction

SINCE 1974, the use of numerical optimization techniques in airfoil design has received considerable attention. The general problem to be solved is stated as

$$\text{minimize } F(\bar{X}) \tag{1}$$

subject to

$$G_j(\bar{X}) \leq 0 \qquad j=1,m \tag{2}$$

where $\bar{X}$ is a vector containing the design parameters which define the airfoil shape. The function $F(\bar{X})$ is the objective function to be minimized, such as the drag coefficient C_D. If it is desired to maximize some function, such as lift, this is done by minimizing the negative of the function; e.g., if the lift coefficient C_L is to be maximized, we minimize $-C_L$. The functions $G_j(\bar{X})$ define constraints which the design must satisfy. These may include, for example, limits on lift, pitching moment, thickness, and camber.

Numerical optimization techniques provide an efficient and versatile tool for the solution of this design problem. A general description of numerical optimization techniques may be found in Ref. 1. In airfoil design applications, the basic approach has been to couple an aerodynamic analysis code with an optimization code to achieve the automated design capability. Most of this work has been directed toward the application of these techniques to a wide variety of design problems while at the same time using increasingly sophisticated and time-consuming analysis programs. Very little effort has been directed toward improving the efficiency of the automated design process. Rather, the principal improvement has been in the method of defining the airfoil. In Refs. 2 and 3, polynomials were used to define the airfoil shape, with the coefficients being the design variables. In Refs. 4 and 5, and in subsequent work, these polynomials were replaced by the use of more general analytical or numerically defined shape functions. The effect of this was an efficiency improvement of more than a factor of 2, together with improved airfoil definition.[4] However, with the large computer times associated with sophisticated analysis programs, major efficiency improvements are still needed if numerical airfoil optimization is to remain an economically feasible design approach.

For purposes of this discussion, efficiency is measured by the number of times the aerodynamics program is called for a complete analysis. This typically accounts for more than 95% of the computer resources. This measure of efficiency is independent of the aerodynamic analysis program and is therefore considered a good measure of design cost, using a given aerodynamics program. A technique is presented here which improves design efficiency by a factor of 2 or more compared with Ref. 4.

To understand the difference in methods, the previous approach to airfoil optimization first will be briefly described. The new design method then will be presented and will be demonstrated by three design examples. A more detailed description of the new method may be found in Ref. 6. Finally, possible future extensions and enhancements will be discussed.

Previous Design Method

Assume that the airfoil is defined by the relationship:

$$\bar{Y}=a_1\bar{Y}^1+a_2\bar{Y}^2+\ldots+a_n\bar{Y}^n \tag{3}$$

where $\bar{Y}$ is a vector containing airfoil upper and lower coordinates and the $\bar{Y}^i$ define shape functions (called basis vectors) which themselves may be airfoils. The coefficients a_1, a_2, ..., a_n are referred to as participation coefficients. These coefficients form the components of the vector of design variables in Eq. (1):

$$\bar{X}=\begin{Bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ a_n \end{Bmatrix} \tag{4}$$

Beginning with an initial design $\bar{X}^0$, a typical numerical optimization program iteratively updates the design such that at iteration $q+1$

$$\bar{X}^{q+1}=\bar{X}^q+\alpha^*\bar{S}^q \tag{5}$$

The vector $\bar{S}^q$ is referred to as a search direction in the n-dimensional design space. The scalar α^* is found by interpolation to yield the greatest design improvement subject

---

Presented as Paper 79-0079 at the AIAA 17th Aerospace Sciences Meeting, New Orleans, La., Jan. 15, 1979; submitted Jan. 18, 1979; revision received June 15, 1979.

Index categories: Aerodynamics; Analytical and Numerical Methods; Configuration Design.

*Associate Professor, Dept. of Mechanical Engineering, Naval Postgraduate School, Monterey, Calif. Member AIAA.

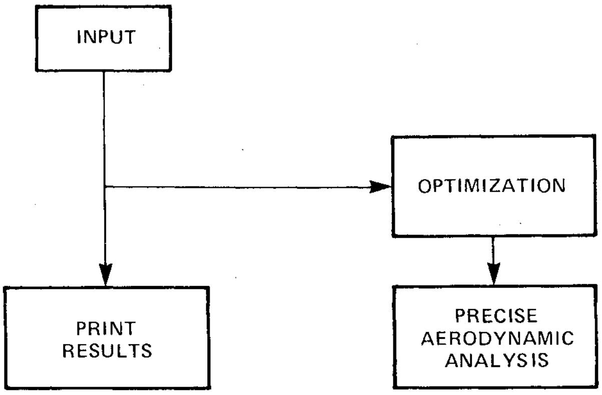

Fig. 1 Previous program organization.

to the inequality constraints. Using gradient-based optimization techniques, determination of $\bar{S}^q$ requires calculation of the gradient of the objective and constraint functions. In airfoil design this is normally done by finite difference, so that n complete analyses are required each time $\bar{S}$ is calculated. Determination of α^* requires (typically) three additional analyses. Present optimization efficiency requires approximately 10 design iterations ($q=1, 2,..., 10$) so that one design requires $10n + 30$ to complete aerodynamic analyses.[1]

At the present time, most airfoil optimization is performed by coupling the aerodynamics program to the optimization program as shown in Fig. 1. Each time the optimization program defines a new design, either for finite-difference gradient computations or for determining α^*, the aerodynamics program is called for a complete analysis. During optimization, at iteration q, very little information from previous iterations is used.

It might be argued intuitively that all calculated information should be of value in guiding the optimization process. Furthermore, in a design study, numerous optimizations are usually performed. For example, one optimization may be done to minimize C_D with constraints on C_L and, later, another optimization done to minimize C_M with constraints on C_L and C_D. It may be expected that, because many airfoils were analyzed during the first optimization, a second optimization at the same flight condition should utilize this available information. One way to do this is to approximate the required functions using available information. This provides explicit functions which can now be optimized independent of the time-consuming aerodynamic analysis program. Aerodynamic analysis is still used to improve the approximation, leading to a precise solution. The general procedure for doing this is outlined in the following section.

New Design Method

The approach used here is to develop a Taylor's series expansion of the various peformance and geometric parameters based on existing data or on data developed earlier in the optimization process. Consider the second-order Taylor's series expansion in matrix form for an arbitrary function:

$$F(\bar{X}) = F^0 + \Delta\bar{X}^T \nabla\bar{F} + \frac{1}{2}\Delta\bar{X}^T [H] \Delta\bar{X} \quad (6)$$

where

$\Delta\bar{X}$ $= \bar{X} - \bar{X}^0$
$\nabla\bar{F}$ = vector of first partial derivatives
$[H]$ = matrix of second partial derivatives (Hessian matrix)
F^0 $= F(\bar{X})$ at $\bar{X} = \bar{X}^0$
$\bar{X}$ = vector of design variables; in this case the coefficients of airfoil basis shapes
$F(\bar{X})$ = approximate function

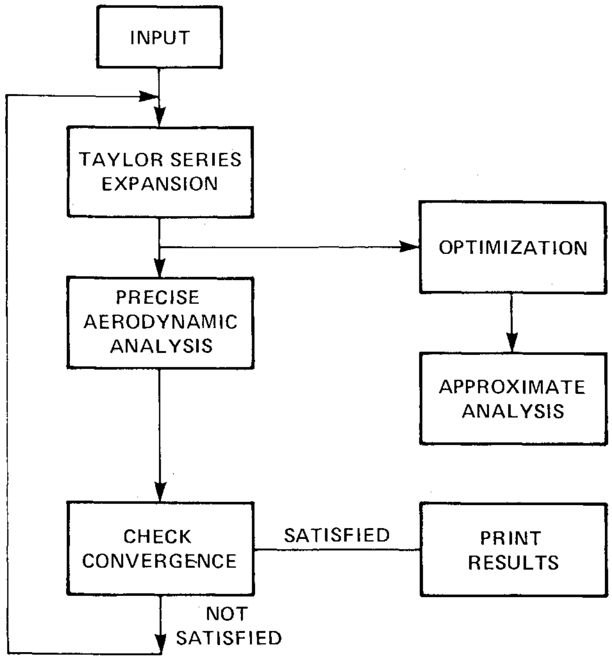

Fig. 2 New program organization.

The function $F(\bar{X})$ is used to denote any pertinent parameter such as C_L, C_D, C_M, t/c, etc.

Assume that $F(\bar{X})$ is known for numerous designs defined by different $\bar{X}^k$. Then the unknowns in Eq. (6) are the components of $\nabla\bar{F}$ and $[H]$. These can be calculated as a set of linear simultaneous equations. If excess data are available, a weighted least-squares fit is used.

Having obtained the approximating functions, they can be used during optimization, rather than calling the aerodynamics program for precise functions. When the optimum airfoil has been found, based on this approximation, the airfoil is analyzed precisely. The results are added to the data set, and new approximating functions are defined. Optimization is then performed using these new functions and the process repeated until convergence is achieved.

The general program organization is shown in Fig. 2. Note that the optimization program never calls the aerodynamic analysis program, but instead optimizes the approximating functions. Therefore, the efficiency of the optimization program itself is of secondary importance because evaluation of the approximate functions as well as the gradient of these functions is quite rapid. It is noteworthy that a similar design approach has been used successfully for structural optimization by Schmit and Miura.[7] Only a second-order approximation is used because higher-order approximations 1) would require excessive data, 2) would tend to model noise in the data, and 3) have been found to be unnecessary.

Earlier work by Moses[8] in structural optimization has utilized sequential linear approximations. This is considered unacceptable for airfoil optimization because the number of binding constraints at the optimum is usually less than the number of design variables, leading to an unbounded solution. This possibility still exists using second-order approximations but is considerably reduced. To insure boundedness of the approximate optimization, bounds are put on the individual design variables to limit the design changes to some reasonable value, say 50% change in airfoil thickness. This insures a solution to the approximate optimization problem while allowing for rapid convergence to the precise optimum.

In practice, the new method begins with only a first-order approximation to the first design variable. This requires two precise aerodynamic analyses to begin the optimization process. Having solved the approximate optimization with respect to a_1, the proposed airfoil is analyzed precisely and the results added to the data set. A first-order approximation is then defined with respect to a_1 and a_2 and the approximate optimization is repeated. This process is continued until at least a first-order expansion with respect to all design variables has been done. Beyond that, the process is continued until the same design has been achieved on two consecutive approximate optimizations, at which point the design process is said to have converged to the solution.

To provide a second-order Taylor series expansion requires $1+n+n\ (n+1)/2$ separate analyses. Therefore, assuming that only a few analyses are required beyond that required for a second-order approximation, the new method will be competitive for designs defined by fewer than 20 design variables. More importantly, if several optimizations are to be performed, data developed as part of one optimization can be used in subsequent optimizations, thus drastically reducing the total number of aerodynamic analyses.

Because arbitrary data are used to calculate the unknown coefficients in Eq. (6) (as compared to small finite-difference perturbations), the Taylor's series coefficients are themselves approximate, becoming increasingly precise as additional data are included in the least-squares fit. The efficiency of this new approach, therefore, hinges on the assumption that the pertinent aerodynamic coefficients can be modeled reasonably well in the region of the optimum by a second-order Taylor's series expansion. In this regard, it should be noted that this is the same assumption on which many of the more powerful optimization algorithms are based.[9] It will be shown by example that this is a good assumption.

Design Algorithm

Having the capability of developing the approximate Taylor's series expansion of the various aerodynamic geometric functions, the expansion is incorporated into an optimization algorithm as follows:

1) Given k initial independent designs, $k \geq 2$
2) Create the Taylor's series expansion about the current "best" design
3) Number of design variables, NDV = min. (k, n)
4) Set limits on the design variables, say $\bar{X}^l = 0.8^* \bar{X}^\circ$ and $\bar{X}^u = 1.2^* \bar{X}^\circ$
5) Optimize the approximating functions
6) Analyze the proposed optimum
7) Add results to data set; set $k = k+1$
8) If $k \leq n+1$ go to step 2
9) Check convergence
10) If satisfied, print final results; otherwise go to step 2

A FORTRAN computer code was written for this technique; a block diagram of the major operations is shown in Fig. 2. The CONMIN program[10] was used for the optimization capability. In the following section, design examples are presented to demonstrate the efficiency of the method.

Design Examples

Examples are presented here to identify the generality and efficiency of approximation concepts as applied to airfoil optimization. Four existing airfoils are used as the design basis vectors in Eq. (3). These are the NACA 2412, NACA 64_1-412, NACA 65_2-415, and the NACA 64_2-A215 airfoils. The coordinates are defined at 50 points along the upper and lower surfaces. Two additional basis vectors are used to impose the geometric boundary conditions at the trailing edge of the airfoil. These are $Y_{us} = X/C$, $Y_{ls} = 0$, and $Y_{us} = 0$, $Y_{ls} = -(X/C)$. The shapes defined by these six basis vectors are shown in Fig. 3. These basis vectors are the same as those used in Ref. 4. For consistency, the same aerodynamic analysis code[11] was also used here. Three of the design

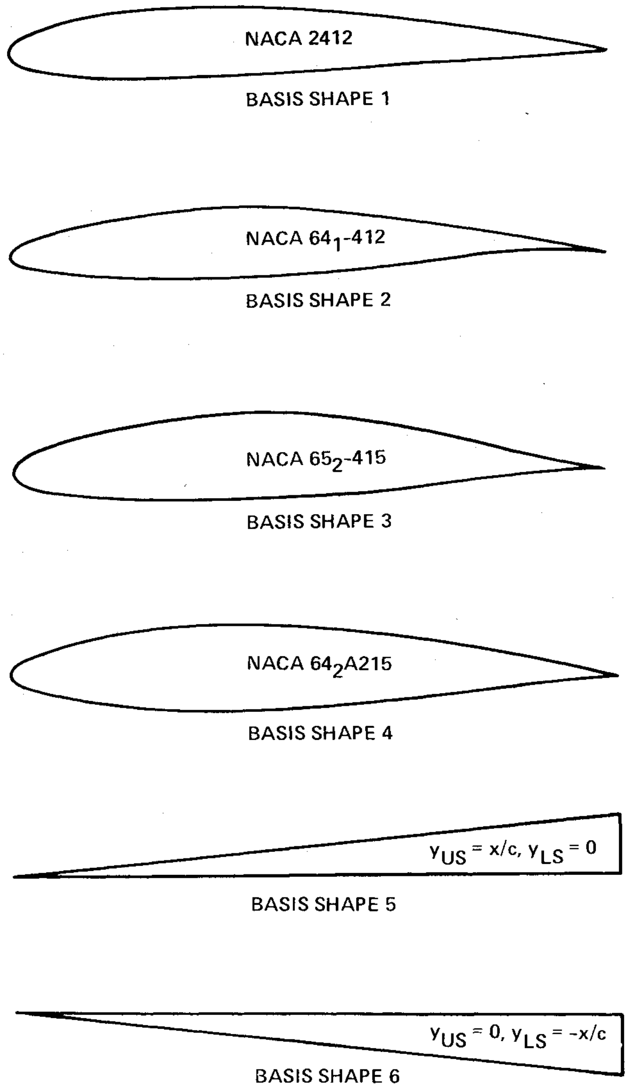

Fig. 3 Basis shapes.

examples of Ref. 4 are solved below; two of these examples were also presented in Ref. 3.

Table 1 compares the efficiency of the present method with that of previous methods. The computer CPU time, on a CDC-7600 computer, for one anlaysis is listed to indicate the cost of the design using the aerodynamic analysis code of Ref. 11. Total CPU time is the CPU time per analysis times the number of analyses.

Example 1: Lift Maximization, $M = 0.1$, $\alpha = 6$ deg

Figure 4 shows the results of optimization of an airfoil for maximum lift. The design constraints are listed on the figure and are the same as those of Refs. 3 and 4. This optimization required 19 aerodynamic analyses, compared to 103 analyses reported in Ref. 3 and 44 analyses reported in Ref. 4. While it may be argued that this airfoil is impractical, it must be remembered that it mathematically satisfies the design constraints. Also, the lift coefficient, $C_L = 1.144$ obtained here is better than the $C_L = 1.106$ obtained before. The fact that this airfoil was not obtained using the previous methods suggests that the present method is numerically better conditioned for optimization. Furthermore, these results were obtained using fewer than half the number of aerodynamic analyses used in Ref. 4. At the optimum, all constraints were critical except the limit on pressure coefficient.

The quality of the approximation to the lift coefficient may be judged from Fig. 5. Because there are four independent

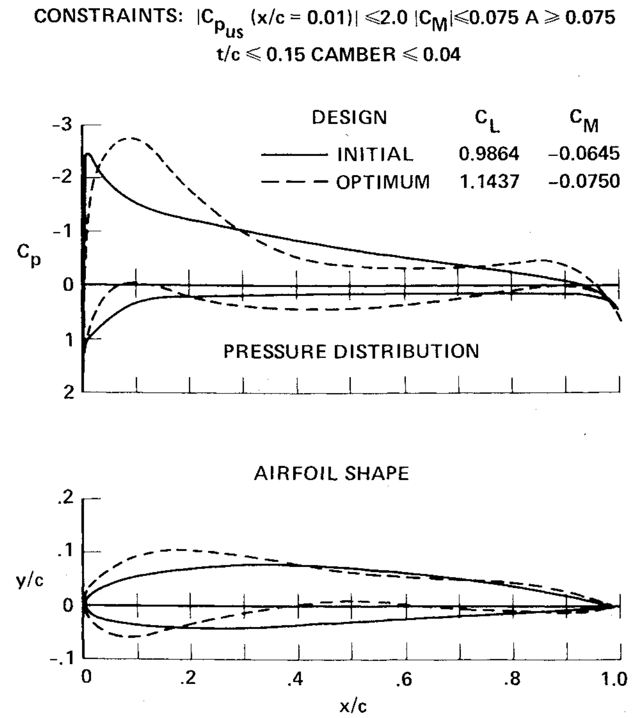

Fig. 4 Example 1: lift maximization $M = 0.1$, $\alpha = 6$ deg.

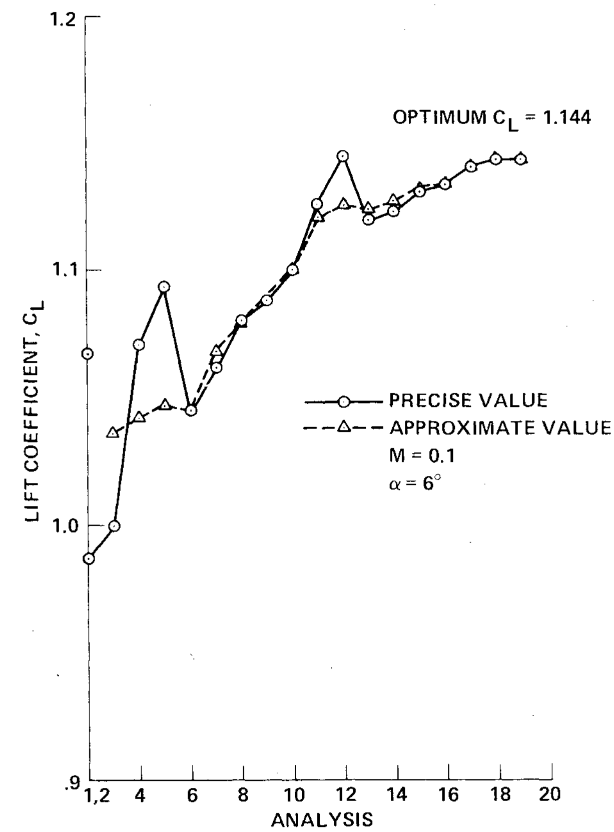

Fig. 5 Optimization history for Example 1.

design variables, the full second-order Taylor's series expansion of the functions requires 15 analyses. It is intriguing to note that on the 16th analysis and beyond, the approximation for this case is quite precise. This suggests that the analysis of low-speed airfoils can be approximated quite well by a second-order Taylor series expansion.

Example 2: Lift Maximization, $M = 0.75$, $\alpha = 0$ deg

Although optimization using sequential approximations works well for low-speed airfoils, it may be expected that the technique would not be adequate for high-speed applications where the nature of the flowfield about the airfoil can be quite sensitive to small changes in the airfoil shape. To study this, examples from Refs. 3 and 4 were solved using the present method. Here, the lift coefficient was maximized subject to a constraint on wave drag. A value of $C_L = 0.4211$ was obtained after 27 analyses, compared to $C_L = 0.3884$ obtained in 143 analyses in Ref. 3, and $C_L = 0.4188$ obtained in 70 analyses in Ref. 4. The results are shown as Example 2A in Fig. 6. Using the present method, the optimization continued to mathematically improve the airfoil and terminated after 48 analyses, yielding the airfoil shown in Fig. 7 as Example 2B. Note the significant changes in pressure distribution between Figs. 6 and 7. The comparison of approximate and precise values of C_L and C_D is shown in Fig. 8. The second-order approximation appears quite good as long as the nature of the aerodynamic flowfield does not change. Difficulty is encountered when design perturbations cause a change in the number of shocks from one to two or two to three. In this design example, the values of C_L and C_D agree well at 27 analyses. The design continues to improve through analysis number 37, at which point designs were considered which had a mild reverse curvature on the upper surface near the leading edge. This results from the inability to properly model a supercritical airfoil using the four NACA basis airfoils. The optimization was able to utilize these data effectively to redirect the optimization process, leading to the final converged solution. At the optimum, both constraints were critical. In a practical design situation, it would be desirable at this point to add other basis vectors that represent supercritical airfoils, remembering that the 48 analyses already obtained provide useful data for the expanded optimization.

Example 3: Wave Drag Minimization, $M = 0.75$, $\alpha = 0$ deg

To demonstrate the efficiency of the present method when multiple optimizations at the same flight condition are performed, a drag minimization example of Ref. 4 was solved. The 48 analyses performed to solve Example 2 here were used as initial data. In Ref. 4, the optimum airfoil from the previous design was used as a starting point for this design. In

Table 1 Design information

Example	Comment	Number of analyses: Ref. 3	Ref. 4	New method	CPU time per analysis, s
1	Initial design	1	1	1	4
1	Optimum design	103	44	19	4
2,3	Initial design	1	1	1	16
2A	After 27 analyses	143	42	27	16
2B	Optimum design	143	70	48	16
3A	Initial analysis No. 27	...	44	2	16
3B	Initial analysis No. 48	...	44	4	16

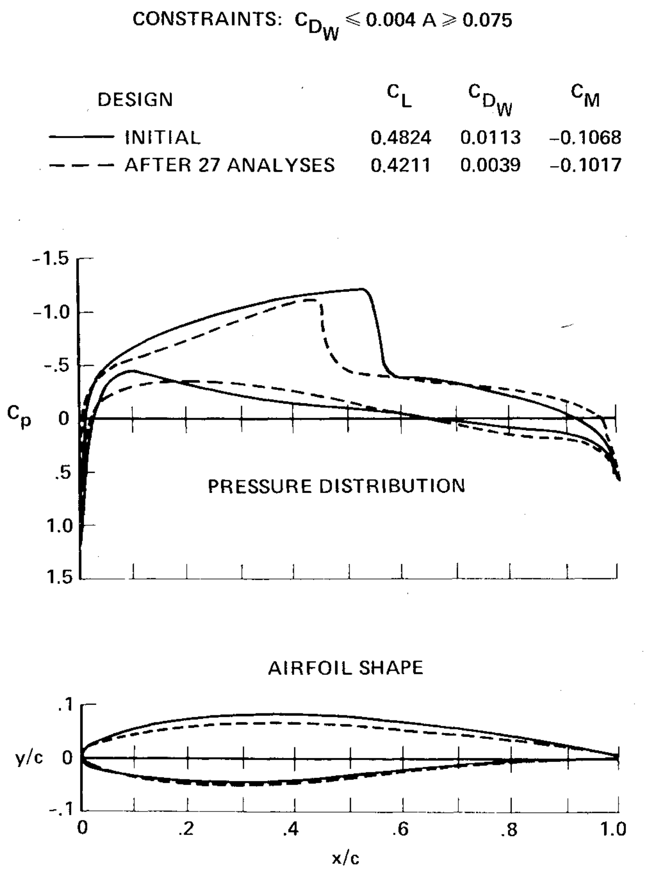

Fig. 6 Example 2A: lift maximization, $M = 0.75$, $\alpha = 0$ deg, design after 27 analyses.

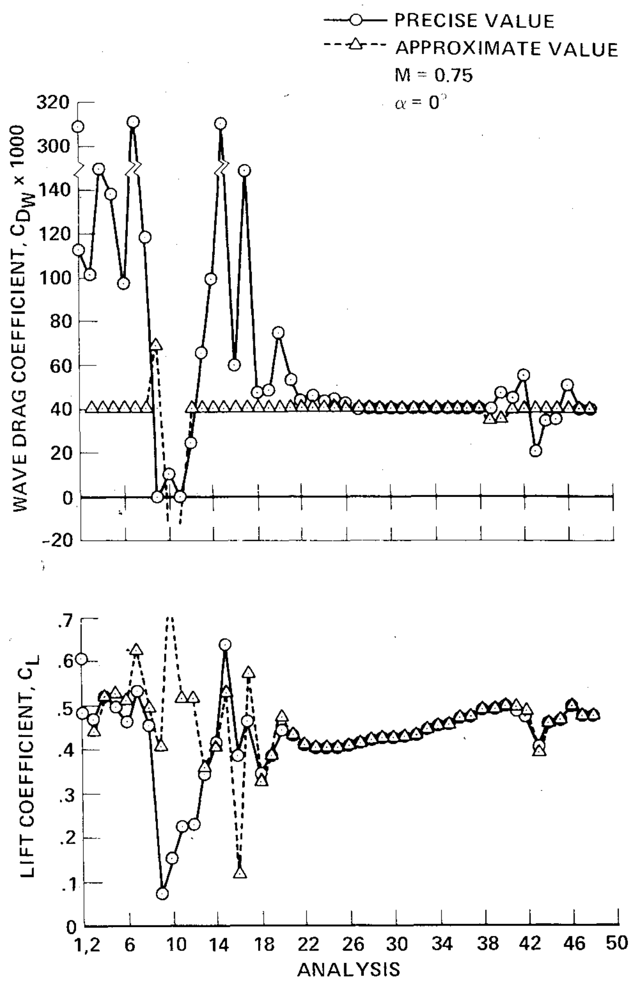

Fig. 8 Optimization history for Example 2.

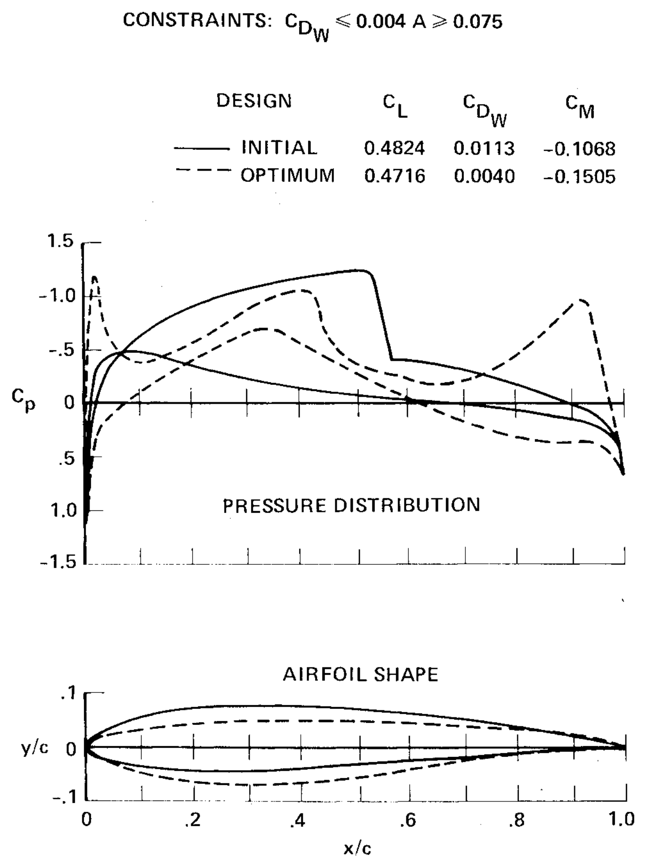

Fig. 7 Example 2B: lift maximization, $M = 0.75$, $\alpha = 0$ deg, optimum design.

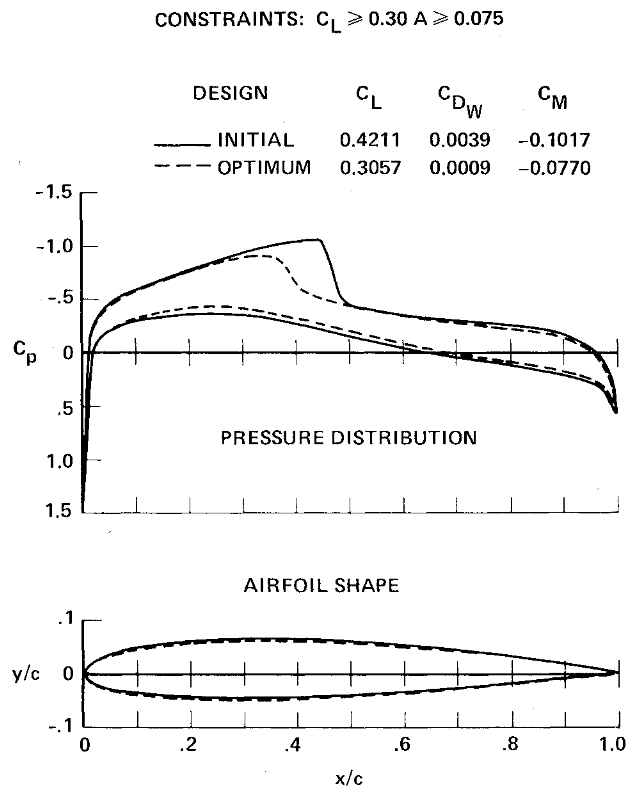

Fig. 9 Example 3A: drag minimization, $M = 0.75$, $\alpha = 0$ deg, beginning with analysis no. 27.

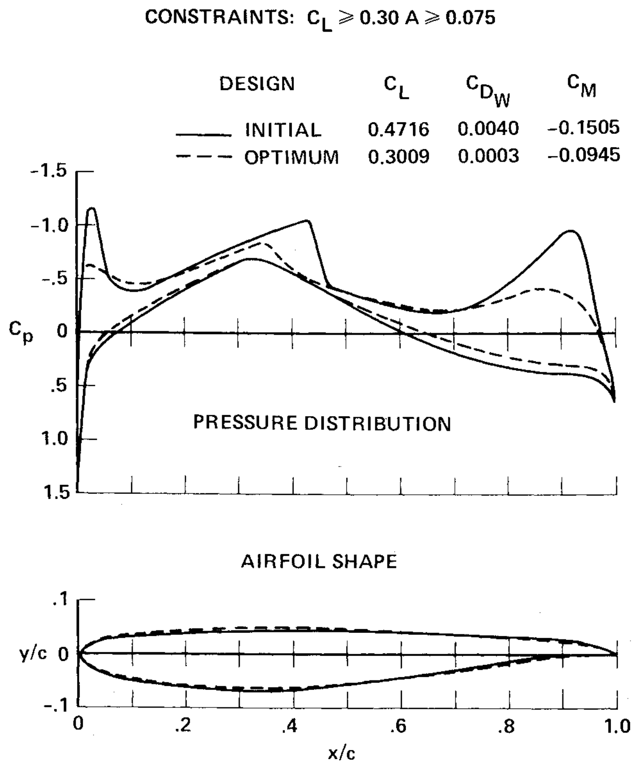

Fig. 10 Example 3B: drag minimization, $M=0.75$, $\alpha=0$ deg, beginning with analysis no. 48.

the present study, the 27th analysis (Fig. 6) was used as the nominal design about which the first Taylor's series expansion was performed. An optimum design of $C_D=0.0009$ was obtained using only two additional analyses. The resulting airfoil is given as Example 3A in Fig. 9 and in Table 1. This result compares to an optimum $C_D=0.0007$ obtained previously using 44 aerodynamic analyses.

As an additional exercise, this design was repeated beginning with the 48th analysis of Example 2 as the initial nominal airfoil. An optimum $C_D=0.0003$ was obtained using 4 additional analyses. This design, Example 3B, is presented in Fig. 10 and in Table 1. As seen from the figures, Examples 3A and 3B represent quite different airfoils, although the actual calculated wave drag is negligible in each case.

Concluding Remarks

An optimization procedure, which efficiently solves the airfoil design problem, has been presented. An important feature of the method is that the sensitivity of the resulting airfoil to small changes in the design variables is automatically provided in the form of the Taylor's series coefficients. In view of the efficiency of this method, further development and extensions are warranted in several cases.

The computer program written for this study is considered a preliminary research program. This will be rewritten as a general-purpose optimization code for public release.

The examples presented here utilized Taylor's series expansions with respect to the geometric sizing variables a_l-a_n. These expressions can just as easily include Mach number and angle of attack. In this way, off-design flight conditions can be included in the optimization process without requiring precise aerodynamic data at every flight condition.

Example 3 demonstrated the extreme design efficiency possible by using existing data for optimization. This motivates the development of design-oriented data storage and retrieval systems so that the ever increasing body of available aerodynamic data can be easily utilized in design. Such a data bank should include experimental as well as analytical data, be easily accessible, and be transportable between computer facilities.

An important extension of the design capability presented here would be to the design of airfoils for specified pressure distribution. One approach is to utilize the second-order Taylor's series expansion of pressure coefficients. Using Newton's method to solve the nonlinear system of equations provides the desired pressure coefficients. The process would be applied repetitively until convergence. The deficiency in such an approach is that the resulting airfoil may be geometrically unrealistic. Future extensions would make use of the numerical optimization techniques to obtain the airfoil most nearly producing the desired pressure distribution while satisfying realistic constraints. The general goal of the work presented here, as well as its proposed extensions, is to develop a distributable computer program and data base which the practitioner can apply to his particular design problem.

References

[1] Vanderplaats, G.N., "The Computer for Design and Optimization," *ASME Winter Annual Meeting,* AMD-Vol. 18, New York, N.Y., Dec. 5-10, 1976.

[2] Hicks, R.M., Murman, E.M., and Vanderplaats, G.N., "An Assessment of Airfoil Design by Numerical Optimization," NASA TM X-3092, 1974.

[3] Vanderplaats, G.N., Hicks, R.M., and Murman, E.M., "Application of Numerical Optimization Techniques to Airfoil Design," NASA SP-347, Pt. II, 1975.

[4] Vanderplaats, G.N. and Hicks, R.M., "Numerical Airfoil Optimization Using a Reduced Number of Design Coordinates," NASA TM X-73,151, 1976.

[5] Hicks, R.M. and Vanderplaats, G.N., "Application of Numerical Optimization to the Design of Supercritical Airfoils Without Drag Creep," *SAE National Business Aircraft Meeting and Exposition,* Wichita, Kansas, March 1977.

[6] Vanderplaats, G.N., "Approximation Concepts for Numerical Airfoil Optimization," NASA TP-1370, 1978.

[7] Schmit, L.A. and Miura, H., "Approximation Concepts for Efficient Structural Synthesis," NASA CR-2552, 1976.

[8] Moses, F., "Optimum Structural Design Using Linear Programming," *Proceedings of the Americal Society of Civil Engineers,* Vol. 90, ST6, 1964, pp. 89-104.

[9] Fox, R.L., *Optimization Methods in Engineering Design,* Addison-Wesley, Mass., 1971.

[10] Vanderplaats, G.N., "CONMIN-A FORTRAN Program for Constrained Function Minimization," NASA TM X-62,282, 1973.

[11] Jameson, A., "Transonic Flow Calculations for Airfoils and Bodies of Revolution," Grumman Aerodynamics, Rept. 390-71, Dec. 1971.